

## ACADEMIC SCHOLARSHIP 2010

## MATHEMATICS

## PAPER 1

$11 / 2$ hours

## CALCULATORS MAY NOT BE USED FOR THIS PAPER.

## INSTRUCTIONS TO CANDIDATES.

You are not expected to have time to do all the questions.
You may answer the questions in any order.
Choose those questions which you think you can answer best.
Remember to show your working and clearly show the method you are using.
Take $\pi$ as either 3.14 or the value on your calculator.
Answers should be given to 3 significant figures where appropriate.
Some questions are longer than others.
The number of marks for each question is shown in square brackets.


1. You are told that $2010=30 \times 67$

Use this fact (without using long division or multiplication) to find the values of:
a) $2010 \div 30$
b) $300 \times 67000$
c) $\quad 0.67 \times 0.003$
d) $2010 \div 15$
e) $2010+67 \times 70$
f) $325 \times 67-175 \times 67$
2. Find all the solutions to the following equations (giving answers either as whole numbers or exact fractions):
a) $4-3 x=5$
b) $2 x^{2}=18$
c) $1+\frac{5}{x}=7$
d) $\frac{x}{2}+\frac{2 x}{3}=4$
e) $\mathbf{< ~}^{2}+x^{7}=\left(-2 x^{7}\right.$,
3. i) Find the values of the following, giving your answers as top heavy fractions:
a) $1+\frac{1}{1+1}$
b) $1+\frac{1}{1+\frac{1}{1+1}}$
c) $1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}$
ii) If we continued to add layers in the same way, use the pattern of your answers to suggest what the next three answers would be.
iii) If the $2010^{\text {th }}$ term is $\frac{a}{b}$, find (with reasons) an expression in terms of $a$ and $b$ for the $2012^{\text {th }}$ term.
4. A circle radius 3 cm is inscribed in a semicircle as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?

5. Of the boys of Maudliner House, $60 \%$ play hockey and $30 \%$ swim. If $40 \%$ of the hockey players swim, what fraction of the non-swimmers play hockey?
6. Mr Ripoff is doing deals in his sweetshop:

## BUY 3 MARS BARS AND 5 TWIXES, PAY $£ 3.80$ BUY 6 MARS BARS AND 2 TWIXES, PAY $£ 3.20$

How much more expensive is the Twix than the Mars Bar?
7. A triangle has sides of length 5,3 and $x$, where $x$ is an integer. Find all the possible values of $x$. Which of these values give triangles containing an obtuse angle?
8. A bag contains 200 counters numbered 1 to 200. A counter is picked at random. Find the probability that it is:
a) a multiple of 5;
b) a multiple of 3;
c) a perfect square;
d) a multiple of 3 or 5;
e) a multiple of 3,4 or 5 .
9. (i) Find the nth term for each of the following sequences:
a) $3,7,11,15,19$, $\qquad$
b) $9,13,17,21,25$, $\qquad$
c) $9,49,121,225,361, \ldots \ldots$.
d) $0,36,104,204,336, \ldots \ldots$
(ii) Does the sequence in part (c) above contain the number $299^{2}$ ?

Explain your answer clearly.
10. a) Sixty-four unpainted $1 \times 1 \times 1$ cubical bricks are put together to form a $4 \times 4 \times 4$ cube. The faces of this large cube are then painted red. Now the cube is dismantled so that some of the 64 bricks have 3 red faces, some have 2 red faces, some have 1 and some have none. How many are there in each category?
b) An $n \times n \times n$ cube is formed by taking $n^{3}$ unpainted bricks as before and again the faces of the large cube are painted red. If the number with one red face is eight times the number with two painted faces, find $n$, the length of the side of the large cube.
11. a) Roads in Oblongville run North-South and East-West only as shown in the grid below:

B


The driver is only able to travel North or East.
i) How many different ways are there to get from $A$ to $C$ ?
ii) How many different ways are there to get from $A$ to $D$ ?
iii) How many different ways are there to get from $A$ to $B$ ?
b) The driver then moves on to Randomcity, shown below. He is still only able to travel North or East.


How many different ways are there for him to get from $S$ to $T$ ?

